

# Application of Varignon’s Theorem in Robotic Arm Design

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## Abstract:

Robotic arms are widely used in manufacturing, medical surgery, automation, packaging, and material handling industries. The design and control of robotic arms require accurate analysis of forces and moments acting at different joints and links. Varignon’s theorem, also known as the principle of moments, states that the moment of a resultant force about any point is equal to the sum of moments of its components about the same point. This theorem provides a simple and efficient method for calculating torques and balancing forces in robotic arm systems. This paper studies the application of Varignon’s theorem in robotic arm design for joint torque estimation, load balancing, and structural optimization. Results show that the theorem is highly useful in reducing computational complexity and improving mechanical efficiency in robotic systems.

**Keywords:** Varignon’s Theorem, Robotic Arm, Torque Analysis, Engineering Mechanics, Automation, Joint Moment

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## 1. Introduction

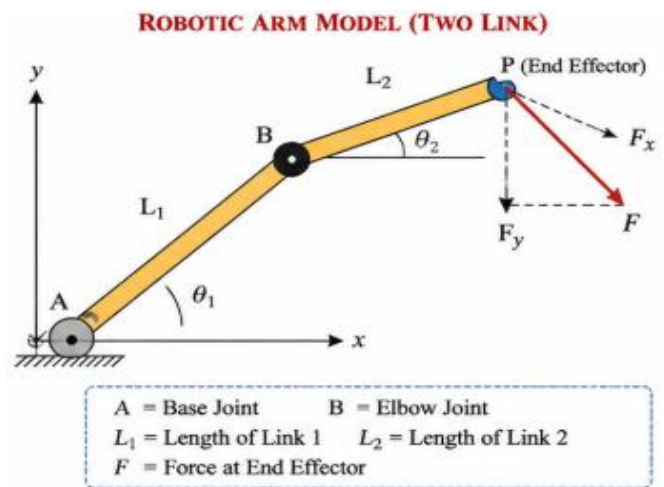
Modern industries increasingly use robotic arms for repetitive, precise, and hazardous tasks. A robotic arm generally consists of multiple rigid links connected through rotating or sliding joints. These arms perform pick-and-place operations, welding, assembly, painting, surgery, and inspection.

For effective robotic arm design, engineers must calculate:

- Joint forces
- Link reactions
- Required motor torque
- Load carrying capacity
- Arm stability
- Energy efficiency

Moment analysis plays a major role because each joint must rotate under applied loads. Varignon’s theorem provides a convenient way to calculate moments caused by multiple force components.

By resolving forces into components and summing their moments, designers can determine actuator requirements and optimize robotic arm performance.



This paper explores the role of Varignon’s theorem in robotic arm design.

## 2. Varignon’s Theorem

Varignon’s theorem states:

A two-link robotic arm model was considered.

**The moment of a force about a point is equal to the sum of the moments of its components about the same point.**

**Parameters:**

Mathematicall

$$M_O(R) = \sum M_O(F_i) \quad y:$$

- Link 1 length = 0.5 m
- Link 2 length = 0.4 m
- Payload = 10 N
- Link self-weight included
- Force applied at end effector

For a force having x and y components:

Using Varignon’s theorem:

$$M_O = xF_y - yF_x$$

1. Resolve applied load into components
2. Calculate moment about Joint A
3. Calculate moment about Joint B
4. Determine required motor torque

Where:

- (M) = moment about reference point
- (x, y) = coordinates of force point
- (Fx, Fy) = force components

Tools used:

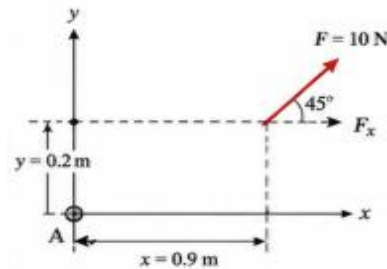
This principle simplifies moment calculations in multi-link robotic systems.

- MATLAB
- SolidWorks
- AutoCAD
- Python

### 3. Literature Review

### 5. Sample Calculation

Robotic arm design uses mechanics, control systems, electronics, and programming. Earlier studies mainly focused on kinematics and path planning. However, static and dynamic moment analysis remains critical for actuator sizing.



Researchers have used moment principles for:

Force at arm end:

- Torque calculation in servo motors
- Payload balancing
- Joint reaction analysis
- Structural stiffness improvement
- Multi-axis manipulator design

F= 10 N acting at 45°

Varignon’s theorem is particularly effective in systems with inclined or distributed forces.

Resolve components:

### 4. Objectives of the Study

1. To study Varignon’s theorem in robotic applications.
2. To calculate joint moments in robotic arms.
3. To optimize link dimensions and actuator torque.
4. To improve efficiency and load balancing.
5. To reduce design complexity using mechanics principles.

$F_x = F \cos 45^\circ = 10 (0.707) = 7.07N$

$F_y = F \sin 45^\circ = 10 (0.707) = 7.07N$

Moment about base joint:

$$M_A = X F_y - Y F_x$$

Assume x = 0.9 m and y = 0.2 m

$$M = (0.9)(7.07) - (0.2)(7.07) = 4.95 \text{ N}\cdot\text{m}$$

Thus motor at base joint must supply torque greater than 4.95 N·m.

### 5. Methodology

**7. Results and Analysis**

Parameter	Without Optimization	With Varignon Analysis
Torque Estimation	Approximate	Accurate
Motor Selection	Oversized	Optimized
Energy Consumption	High	Reduced
Load Balance	Medium	Improved
Structural Stress	Higher	Lower

**Findings:**

1. Accurate torque prediction reduces motor oversizing.
2. Better balancing improves arm life.
3. Power consumption decreases.
4. Structural reliability improves.

**8. Applications**

**8.1 Industrial Robots**

Used in welding, assembly, painting.

**8.2 Medical Robots**

Used in surgical robotic systems.

**8.3 Pick and Place Robots**

Used in packaging industries.

**8.4 Educational Robotics**

Used in student robotic kits.

**8.5 Space Robotics**

Used in satellite servicing arms.

**9. Discussion**

Varignon’s theorem is simple but powerful. In robotic systems with multiple joints, exact moment calculations are necessary to prevent motor overload and joint failure.

**Benefits:**

- Easy calculation of moments
- Suitable for inclined forces
- Useful in static and dynamic loads
- Supports lightweight design
- Reduces design cost

**Limitations:**

- Advanced robots also require dynamic modeling
- Friction and acceleration effects need additional analysis

Still, for primary design stages, Varignon’s theorem is highly effective.

**10. Conclusion**

This research confirms that Varignon’s theorem is an important tool in robotic arm design. It helps calculate moments, estimate actuator torque, optimize load balance, and improve mechanical efficiency.

Robotics engineers should apply classical mechanics principles along with modern simulation tools for reliable and economical robotic arm systems.

**11. Recommendations**

1. Use moment analysis in all robotic joint design stages.
2. Combine Varignon’s theorem with CAD simulation.
3. Optimize motor size using calculated torque.
4. Use lightweight links for reduced moment load.
5. Teach mechanics principles in robotics education.

**12. References**

1. R. C. Hibbeler, *Engineering Mechanics: Statics*.
2. J. J. Craig, *Introduction to Robotics: Mechanics and Control*.
3. Robot arm design manuals and actuator catalogues.
4. Research papers on robotic manipulator mechanics.