

# State and Output Feedback Control Strategies for Non-linear Systems: Design and Implementation

Sanjay Bhadra

Dept. of Electrical Engineering IEM,  
NewTown Campus UEM, Kolkata  
Kolkata, India

**Abstract**—Nonlinear process control is an important tool in process industry. In this work the authors have proposed a nonlinear model-based control scheme. The paper examines the servo and regulatory responses of the proposed approach. Extensive simulation results demonstrate that the controller provides improved performance compared to conventional PI schemes, while effectively reducing measurement noise and enhancing robustness.

**Keywords**—NMBC, State and output feedback, Conventional PI control.

## I. INTRODUCTION

Because of the nonlinear dynamic behaviour, time varying uncertain parameters, interaction between manipulated and controlled variables, unmeasured and frequent disturbances and limits on manipulated variables, chemical processes pose a number of difficult issues. The majority of chemical process industries require conventional control techniques due to their inherent nonlinearity. However spherical tanks are often used in gas plants. Because their area of cross section constantly changes as the tank's height does so, they are nonlinear systems. One extremely powerful structure is a spherical. There are no weak spots since the pressures on the sphere's surfaces are distributed evenly. Additionally compared to the other vessel shapes they have a lower surface area per unit volume. Thus less heat will be transported from warmer surroundings to the liquid in the sphere than in cylindrical or rectangular storage vessels, resulting in less pressure from external influence. Controlling a spherical tank is crucial since the nonlinearity arises from shift in shapes. Numerous industries employ the proportional-integral (PI) and proportional integral derivative (PID) control modes.

However PID and non-linear model based control schemes (NMBC) are the most frequently used control schemes over past few decades. As inherent nonlinearity is prevalent in most of the real time systems hence non-linear control methodologies would be a better choice in controlling non-linear plant variables rather than typical linear controllers, which requires re-linearization in subsequent stages. In case of adaptive non-linear system, process states need to be updated with respect to time to cope up with process uncertainty [1].

Due to difficulties of achieving exact values of system parameters from the real plant, a non-linear model based

control (NMBC) law needs to be employed for the convergence with true process parameter value [1]. As a well recognized example of feedback control loop, NMBC scheme was improvised for non linear process. However, NMBC control scheme specifies output/state variable feedback to formulate driving action for generating proper manipulated variable to control the system output [1]. The foremost drawback of designing NMBC controller faces real problem especially in case of sudden changes in process parameters or load disturbances [1,17]. Hence, model structure is required to balance with associated uncertainty with process or even in case of disturbances in load. A typical structural characteristics based models have been studied and considered in this proposed work.

Thus, the objective of this study is to observe the performance by incorporating adaptation mechanism in NMBC control scheme [5-8]. In this proposed work, the authors have implemented combination of state feedback and output feedback based control schemes. A first principle NMBC control scheme has been considered in this work. A non-linear model has been incorporated to compute process gain, which in turn calculates controller output to obtain desired output. The motivation of the proposed control schemes lead to development of an accurate non-linear controller which would be able to track desired value and would be able to eliminate load disturbances. Performance of the proposed scheme shows significant improvement in uncertainties associated with process/load disturbance.

## II. CONTROL SCHEMES

### II.I. CONVENTIONAL ADAPTIVE PI CONTROL SCHEME (CA-PI)

The adaptive conventional PI control law has been described by the general equation as [1,14,21]:

$$u(k) = u(k-1) + (k_{c1} * \partial e_1(k)) + (T_s * (k_{c1} / T_i) * e(k)) \quad (1)$$

Where  $k_{c1}$  and  $T_i$  are proportional gain and integral time constant of the conventional adaptive PI control scheme respectively.  $T_s$  is the sampling time.  $e$  is the process error which can be described as:

$$e(k) = y_{sp}(k) - y(k) \tag{2}$$

$$\partial e(k) = e(k) - e(k-1) \tag{3}$$

Where  $(k_{c1}; T_i)$  is the conventional adaptive PI controller tuning parameter. Conventional adaptive PI controller has been named as CA-PI shortly.

**II.II. PROPOSED CONTROL SCHEMES**

An adaptive non-linear model based control scheme implemented on spherical tank processes have been proposed here. The proposed control law for the above mentioned systems are as follows:

$$u(k) = \alpha * k_{c2}(k)(y_{sp}(k) - y(k)) + u_d(k) \tag{4}$$

Where  $y_{sp}$  is the set-point.  $y_m$  is the model state. Controller gain  $k_{c2}$ , bias ( $u_d$ ) and numerical multiplier ( $\alpha$ ) are considered to be the tuning parameters of the proposed control scheme.

Controller gain  $k_{c2}$  can be derived in different ways:

**A. Proposed control scheme1 (PS1):**

Different operating regions  $(\bar{u}, \bar{x}, \bar{y})$  can be obtained from open loop studies. A mathematical abstraction between input and output can be formulated based on various operating regions. It was assumed that a linear relation exists between input and output. Hence  $k_{c2}$  can be derived as:

$$k_{c2} = f(\bar{u}, \bar{y}) \text{ where } k_{c2} \text{ has linear function of } (\bar{u}, \bar{y}) \tag{5}$$

The schematic diagram of the proposed control scheme is shown in Fig.1.

**B. Proposed control scheme2 (PS2):**

Similarly like PS1, a mathematical deduction between input and output can be formulated based on various operating points, where it was assumed that a non-linear relation exists between input and output. Hence  $k_{c2}$  can be derived as:

$$k_{c2} = f(\bar{u}, \bar{y}) \text{ where } k_{c2} \text{ has a quadratics non-linearity of } (\bar{u}, \bar{y}). \tag{6}$$

The schematic diagram of the proposed control scheme is shown in Fig.1.

**C. Proposed control scheme3 (PS3):**

In this PS3  $k_{c2}$  can be derived as:

$$k_{c2}(k) = u_d(k) / y_m(k) \tag{7}$$

The schematic diagram of the proposed control scheme is shown in Fig.2.

**III. PROCESS DESCRIPTION AND SIMULATION RESULTS**

The processes considered for the simulation study is spherical tank.

**III.I. SPHERICAL TANK SYSTEM**

The transient material balance equation for the spherical tank system is as follows [9,21]:

$$A(h) \frac{dh}{dt} = f_{in} - f_{out} \tag{8}$$

Where  $A(h)$  is the area of the tank.  $f_{in}$  (cc/sec) and  $f_{out}$  (cc/sec) are inflow and outflow rates of the tank respectively.  $h$  (cm) is the water level in the spherical tank system. The relationship between outflow rate ( $f_{out}$ ) and the water level ( $h$ ) in the tank is as follows:

$$f_{out} = c_v \sqrt{2gh} \tag{9}$$

Where,  $c_v$  is the discharge co-efficient. It should be noted that the area of the spherical tank varies with water level in the tank. In case of spherical tank, the area of the tank can be calculated using following relations.

$$A(h) = \pi \left[ 2rh - h^2 \right] \tag{10}$$

Where,  $r$  is the maximum radius of spherical tank.

All the simulations were executed by considering first principle model mentioned in equation (10-12). True state variable is obtained by solving differential equation using MATLAB 7.2 toolboxes. In the entire simulation study, sampling time has been considered as 0.0833 min. Table I. mentions the process parameter data for spherical tank. A constrained on the manipulated variable ( $0.01 < f_{in} < 20cc/sec$ ) has been imposed. Following operating point has been taken for entire simulation study ( $\bar{h} = 30, f_{in} = 2.7386$ ).

**III.I.I. OPEN LOOP STUDY**

In order to assess the open loop study of the spherical tank system, a sequence of step changes (combination of positive and negative steps) in the manipulated variable has been introduced. Figure 3(a) represents the step-changes in the input ( $f_{in}$ ). The variation of process output ( $h$ ) is reported in Fig.3(b).

**III.I.II. SERVO RESPONSE**

In order to assess the tracking capability of all the above mentioned control schemes, set point variation as shown in Fig.4(b) has been introduced. Simulation studies were made based on the set point variation from 30 to 35 and finally from 35 to 25. From Fig.4(b), it can be observed that the all the controllers are able to maintain the set point at desired level. The variation of controller output is reported in Fig.4(c). The evolution of the CA-PI controller tuning parameters ( $k_{c1}, T_i$ ) and proposed controller tuning parameters ( $k_{c2}, u_d$ ) have been shown in Fig.5. From the study, it can be concluded that CA-PI control scheme is oscillatory in nature. However, it can

be seen that controller is having poor robustness. The robustness of both the proposed controllers was found to be satisfactory. In order to achieve desired level quickly at the time of set point variation, controller tuning parameter  $[\alpha]$  need to be increased. From the observations, it was found that, significant increase of  $[\alpha]$ , robustness of the controller would have negligible impacts.

### III.I.III. REGULATORY RESPONSE

In order to assess the disturbance rejection capability of all the mentioned control schemes, step like changes in the downstream valve position as shown in Fig.4(a) has been introduced. In this study, set point was maintained at 30 cm. It should be noted that a positive step change in the downstream valve position of magnitude 0.05 (from 0.5 to 0.55) has been introduced at 101th sampling instance and the same value has been maintained up to 500th and again a negative step change in the downstream valve position of magnitude 0.05 (from 0.55 to 0.5) has been introduced at 501th sampling instance and the same value has been retained up to 1000th sampling instance. From Fig.4(d), it can be observed that all the controllers are able to reject disturbances and bring the process variable back to the desired level. The variation of controller outputs is reported in Fig.4(e). The evolution of CA-PI controller tuning parameters ( $k_{c1}, T_i$ ) and proposed controllers tuning parameters ( $k_{c2}, u_d$ ) have been shown in Fig.5. From the study, it can be concluded that eliminating disturbances with CA-PI control scheme is oscillatory in nature which takes longer time to settle. It can be inferred that the controller is having poor robustness. The merits of the proposed schemes also lead to the robustness feature of the controller.

### IV. CONCLUSION

From the extensive simulation study, it can be concluded that the servo and regulatory performances of the proposed control schemes applied to the spherical tank process are satisfactory. Comparative analysis indicates that the proposed schemes achieve faster settling times than the CA-PI controller during set-point variations. It is also observed that the proposed schemes eliminate disturbances more quickly compared to the CA-PI control scheme. Furthermore, the CA-PI controller exhibits poorer robustness in comparison to the proposed approaches. The proposed control schemes are also more effective in reducing measurement noise than the CA-PI controller. Overall, based on performance evaluation, the proposed control schemes outperform the CA-PI control strategy.

### REFERENCES

- [1] Bequette BW (1991) Nonlinear control of chemical processes: processes: a review. *Ind Eng Chem Res.* 30(7): 1391-1413.
- [2] Astrom K.J. and T. Haggglund (1988), Automatic tuning of PID controllers, Instrument society of America, Research Triangle Park, NC.
- [3] Economou CG (1986), An operator theory approach to nonlinear controller design, Ph.D. Dissertation, California Institute of Technology, Pasadena.
- [4] Francis JD, Babatunde A. Ogunnaik and Ronald K. Pearson (1995) Nonlinear Model-based Control Using Second-order Volterra Models. *Automatica.* 31(5): 697-714.
- [5] Qiuping Hu, Gade Pandu Rangaiah (1999) Adaptive internal model control of nonlinear processes. *Chemical Engineering Science.* 54(9): 1205-1220.
- [6] Edgar CR, B. E. Postlethwaite (2000) MIMO fuzzy internal model control. *Automatica.* 34(6): pp. 867-877.
- [7] Ling Wei-Ming, Daniel E. Rivera (2001) A methodology for control-relevant nonlinear system Identification using restricted complexity models. *Journal of Process Control.* 11(2): 209-222.
- [8] Azlan HM, Paisan Kittisupakorn and Wachira Daosud (2001) Implementation of neural-network-based inverse-model control strategies on an exothermic reactor. *Science Asia.* 27: 41-50.
- [9] Tan KK, Huang S and Ferdous R (2002) Robust self - tuning PID controller for nonlinear systems. In: *The 27th Annual Conference of the IEEE Industrial Electronics Society (IECON 2001)*, Denver, CO, USA, 29 November - 2 December 2001, pp. 758-763: IEEE Publishing.
- [10] Michael PN, Costas Kravarisb (2003) Nonlinear model-state feedback control for non minimum - phase processes. *Automatica.* 39(7): 1295-1302.
- [11] Findeisen R, Lars Inslund, Frank Allgower and Bjarne A. Foss (2003) State and Output feedback Nonlinear Model Predictive Control: An Overview. *European Journal of Control.* 9(2-3): 190-206.
- [12] Galan O, Josh A. Romagnoli and Ahmet Palazoglu (2004) Real-time implementation of multi-linear model - based control strategies - An application to a bench-scale pH neutralization reactor. *Journal of Process Control.* 14(5): 571-579.
- [13] Padmasree R, Srinivas M.N. and Chidambaram M. (2004) A simple method of tuning PID controllers for stable and unstable FOPTD systems. *Computers and Chemical Engineering.* 28(11): 2201-2218.
- [14] Shamsuzzoha AM. and Lee M. (2007) IMC-PID Controller for improved disturbance rejection of time delayed process. *Ind. Eng. Chem.Res.* 46(7): 2077-2091.
- [15] Deng Hua, Zhen Xu and Han-Xiong Li (2009) A novel neural internal model control for multi-input multi-output nonlinear discrete-time processes. *Journal of Process Control.* 19(8): 1392-1400.
- [16] Prakash J. and K. Srinivasan (2009) Design of nonlinear PID controller and non linear model predictive controller for a continuous stirred tank reactor. *ISA Transactions.* 48(3): 273-282.
- [17] Arun R P and Jagadeesan Prakash (2014) Design and implementation of a model based PI like control scheme in a reset configuration for stable single loop systems. *The Canadian Journal of Chemical Engineering.* 92(9): 1651-1660.
- [18] Firdaus E. Udwardia (2014) A New Approach to Stable Optimal Control of Complex Nonlinear Dynamical Systems. *Journal of Applied Mechanics.* 81: 031001-1 to 031001- 6. *Chemical Engineering.* 92(9): 1651-1660.
- [19] Liu Tengfei and Zhong Ping Jiang (2015) Event-based control of nonlinear systems with partial state and output feedback. *Automatica.* 53: 10-22.
- [20] Atanu Panda and Jagadeesan Prakash (2017) State Estimation and Non-Linear Model based Control of a Continuous Stirred Tank Reactor Using Unscented Kalman Filter. *The Canadian Journal of Chemical Engineering.* 95(7): 1323-1331.

Table I Values of the process parameters & variables associated with spherical tank system

| Process variable            | Nominal operating value |
|-----------------------------|-------------------------|
| Maximum Height (H)          | 60 cm                   |
| Maximum Radius (R)          | 25 cm                   |
| Valve Coefficient ( $C_v$ ) | 0.5                     |

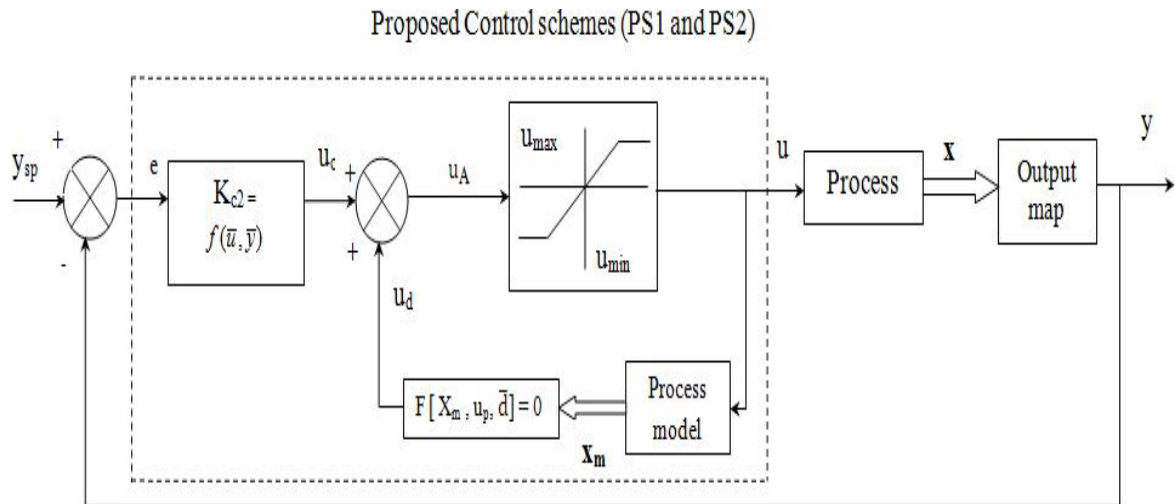


Fig.1. Schematic diagram of proposed control schemes (PS1 and PS2).

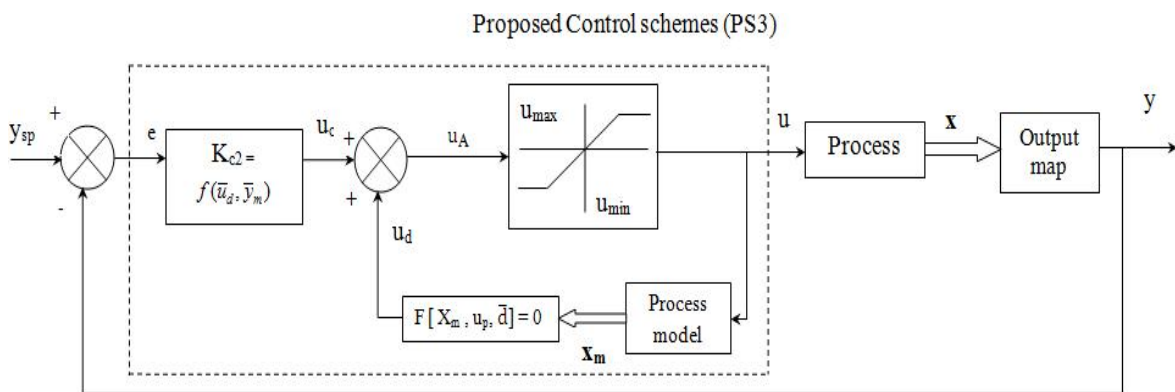


Fig.2. Schematic diagram of proposed control schemes3 (PS3).

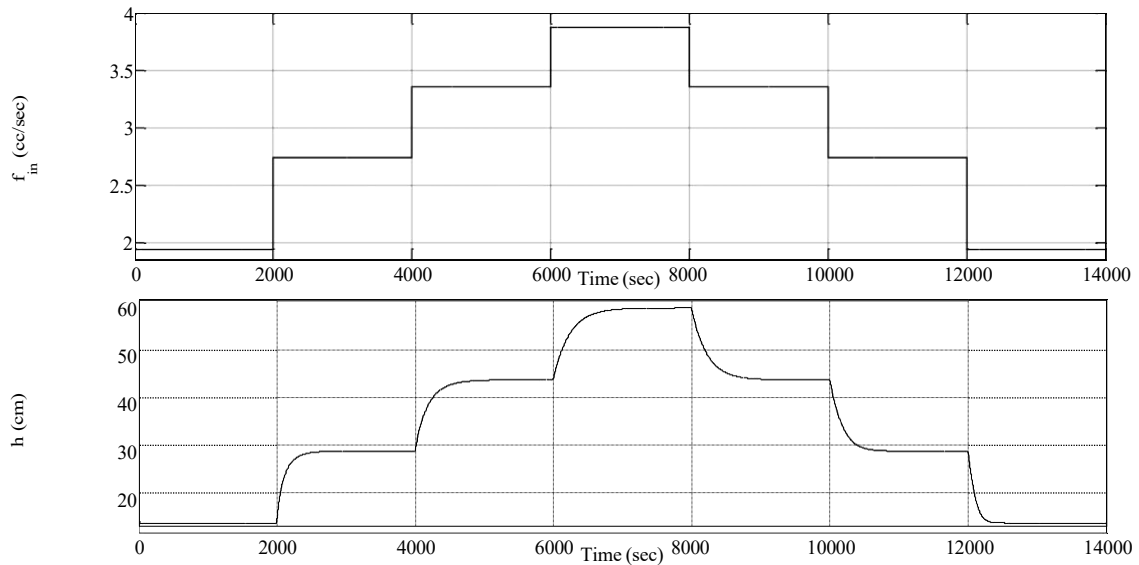


Fig.3. Open loop response of the spherical tank processes (a) variation of input, (b) process output.

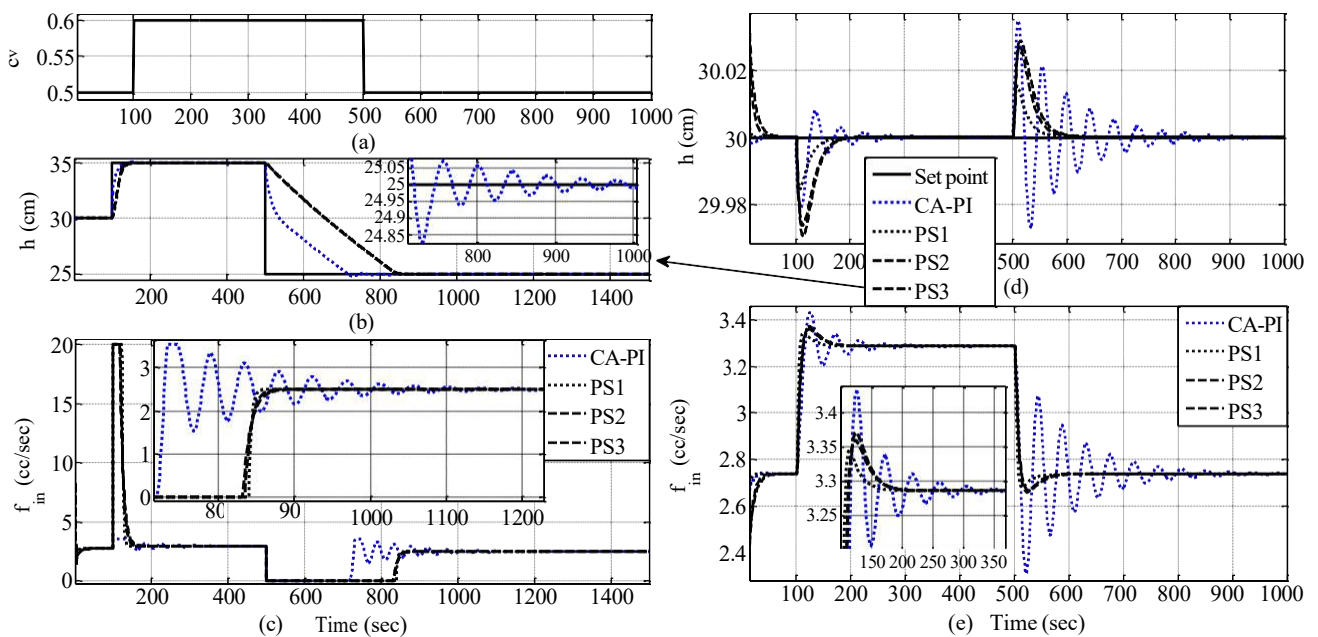


Fig.4. Servo and regulatory response of the spherical tank process with various control schemes (a) Evolution of downstream valve ( $c_v$ ), (b) process output – servo response, (c) variation of controller output – servo response, (d) process output – regulatory response, (e) variation of controller output – regulatory response.