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Novel Characteristics Evaluation of Critically Damped Second-order-like Dynamic Systems

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Abstract:

This paper is presents a novel evaluation procedure for the settling time, delay time and rise time as important time-based characteristics of critically damped second-order-like dynamic systems with 0/2 and 1/2 orders. The time-based characteristics are defined by simple analytical model giving the characteristic as proportional to the reciprocal of the system natural frequency for 0/2 systems. Polynomial models is fitted for each characteristic against the time constant of the simple zero of the 1/2 dynamic system. The evaluation procedure finds optimal values for the 1/2 dynamic system natural frequency. The feasibility of the proposed approach is examined using two case studies where the time-based characteristics are compared (exact and present) with statistical measures.

Keywords — Critically damped second-order-like dynamic systems, Step time response, Step time response characteristics, 0/2 second-order systems, 1/2 second-order systems.

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I. INTRODUCTION

A lot of industrial processes and other applications can be dynamically classified as second-order-like processes. Because critically damped second order dynamic systems have no overshoot and fast step time response, this research work is concentrated only on the characteristics of critically damped second-order systems. We start by presenting a simple literature review about the subject since 2004 :

MIT (2004) investigated the transient time response of first and second-order systems. They presented the unit step time response of second-order control system having damping ratio from 0.1 to 0.707(underdamped), 1 (critically damped and 1.5-5 (overdamped). They did not present any mathematical expressions for the characteristics of the second-order step time response [1]. Angelele (2011) investigated the time response of first and second-order dynamic systems with special concentration on mechanical systems starting with impulsive time response, then ramp time response and step time response. He did not refer to the time-based characteristics of the step time response of second order systems [2]. Swarnkar, Jain and Nema (2011) used a model reference adaptive controller with adaptation gain to control a second-order process having 0.163 damping ratio and 24.5 rad/s natural frequency providing 60 % maximum overshoot without control. They did not provide any analytical relationships for the time-based characteristics of the second-order process step response [3]. Paja, Gonzalez and Mentes (2013) proposed a procedure to accurately calculate the settling time of second-order systems for any damping ratio and natural frequency. They outlined that some authors proposed using explicit equations to calculate the settling time which can introduce significant errors depending on damping ratio and natural frequency of the second-order system. They presented the equations given by Ogata [5] and Kue [6] for underdamped second-order systems [4].

El-Hussieny (2016) analyzed the step time response of second-order systems. He presented analytical relationship for the settling time as function of system damping ratio and natural frequency for underdamped systems with 2% tolerance. He did not handle the cases of critical and overdamped systems [7]. Rachides (2017) investigated the transient time response of first and second-order systems. He presented the transient response specifications of underdamped second-order systems including equations for the rise time, peak time and settling time [8]. Mustansiriyah University (2020) analyzed a single-loop block diagram producing a standard 0/2 second-order control system. They presented the unit step time response of the 0/2 second-order system for damping ratio of 0 to 0.8 (underdamped), 1 (critically damped) and 2 (overdamped). They presented the transient response specifications for underdamped second-order systems. They derived analytical equations for the rise time, peak time , maximum overshoot and settling time for underdamped systems [9].

Babu et al. (2021-2022) investigated the time response analysis of control systems including that of a second-order systems. They provided mathematical equations for the delay time, rise time, peak time, maximum overshoot and settling time for 2% and 5 % tolerance for underdamped second-order systems [10]. Dorf and Bishop (2022) in their book about 'modern control systems' presented the performance of second-order systems where they presented the time step time response for damping ratio from 0.1 to 0.7 (underdamped), 1 (critical damping) and 2 (overdamped) systems. They presented equations for the settling time, peak time, maximum overshoot and rise time for underdamped second-order systems [11]. Stacco (2023) used a closed-form metrics of normalized second-order system to derive a simple design procedure to identify second-order approximation with most relevant dynamic characteristics of the target system. They applied their approach to underdamped and overdamped second-order systems from its step response. They presented equations for the peak time, settling time, damping ratio for a specific natural frequency, maximum overshoot and rise time for underdamped systems. [12].

Cheung (2024) investigated the step time response of second-order systems of the 0/2 type and presented their step time response for damping ratio of: 0.1, 0.2, 0.5, 1, 2 and 4. He presented a unit step time response for 0.2 damping ratio and 0.25, 0.5, 1, 2 and 4 natural frequency. He did not present any mathematical expressions for the time-based characteristics of the second-order systems [13]. Malczyk (2025) investigated the properties and performance of second-order systems and presented the unit step time response for damping ratio in the range 0-2 (underdamped, critical damping and overdamped). For underdamped second-order systems he presented equations for the rise time, peak time, maximum overshoot and settling time [14.].

II. CRITICALLY DAMPED 0/2 SECOND-ORDER DYNAMIC SYSTEM

A lot of dynamic systems exhibit 0/2 overdamped second-order dynamic model characteristics in industrial engineering [15], mechanical engineering [16], aeronautical engineering [17], automotive engineering [18], locomotive engineering [19], civil engineering [20] and biomedical engineering [21]. The transfer function of a 0/2 second-order dynamic system , $G_{ds1}(s)$ is given by [5], [6], [11]:

$$G_{ds1}(s) = \omega_n^{\,2} / (s^2 + 2\varsigma\omega_n s + \omega_n^{\,2}) \qquad (1)$$

Closed-form equations for the settling time and other characteristic functions are covered for underdamped second-order systems by a large number of authors (e.g. [1], [5], [6], [7], [8], [9], [10], [11], [12] and [14]. Critically damped second-order systems have outstanding characteristics including zero maximum overshoot and minimum settling time. Those characteristics are very useful in tuning some controllers or compensators for first and second-order processes. A critically damped dynamic system has a unit step time response, $c_1(t)$ given by [14]:

$$c_1(t) = 1 - (1 + \omega_n t)\exp(-\omega_n t) \qquad (2)$$

Settling Time, T_s:

The settling time of a step time response of a dynamic system is the time after which the time response settles within a ± 2% tolerance. With critically damped dynamic systems, this reduces to the intersection of the unit step time response with a 0.98 line. At t = T_s, Eq.2 gives:

$$(1+\omega_n T_s)\exp(-\omega_n T_s)-0.02=0 \qquad (3)$$

Eq.3 is a nonlinear equation in the dynamic system settling time. For a specific value of the system natural frequency, it is solved by the MATLAB command '*fsolve*' [22]. The application of '*fsolve*' to solve equation 3 (giving the settling time T_s) for natural frequency, ω_n in the range $1\leq \omega_n \leq 10$ rad/s. The results are given in Table I with the parameter K_{Ts} ($T_s\omega_n$).

TABLE I
SETTLING TIME OF THE 0/2 CRITICALLY DAMPED SECOND-ORDER DYNAMIC SYSTEM

ω_n (rad/s)	T_s (s)	K_{Ts}
1	5.8340	5.8340
2	2.9200	5.8400
3	1.9446	5.8338
4	1.4584	5.8336
5	1.1668	5.8701
6	0.9783	5.8701
7	0.8335	5.8345
8	0.7300	5.8400
9	0.6483	5.8343
10	0.5835	5.8350

ω_n: System natural frequency
T_s: System settling time for 2% tolerance.
K_{Ts}: Settling time gain in: $T_s = K_{Ts}/\omega_n$

It is obvious from Table I that the gain K_{Ts} has very close values for the ω_n range investigated. The '*mean*' command of MATLAB [23] is used to provide the mean value of the gain KTs and the '*std*' command [24] is used to provide its standard deviation about its mean value. The two parameters for K_{Ts} are as follows:

$$K_{Ts-mean}=5.8355, St-Deviation_{-Ts}=0.00259 \qquad (4)$$

Rise Time, T_r:

The rise time of a step time response of a dynamic system is the time after which the time response rises from 10 % to 90 % of its steady-state value [11] . With critically damped dynamic systems, this reduces to the intersection of the unit step time response with a 0.10 line for T_{r1} and 0.90 line for T_{r2} where the rise time T_r will be T_{r2}-T_{r1} given from the two equations:

For T_{r1}:

$$(1+\omega_n T_{r1})\exp(-\omega_n T_{r1})-0.9=0 \qquad (5)$$

For T_{r2}:

$$(1+\omega_n T_{r2})\exp(-\omega_n T_{r2})-0.1=0 \qquad (6)$$

Eqs.5 and 6 are nonlinear equations in the dynamic system rise time. For a specific value of the system natural frequency, it is solved by the MATLAB command '*fsolve*' [22]. The application of '*fsolve*' to solve Eqs.5 and 6 (giving the rise time elements T_{r1} and T_{r2}) for natural frequency, ω_n in the range $1 \leq \omega_n \leq 10$ rad/s. The results are given in Table II with T_r=T_{r2}-T_{r1} and the parameter K_{Tr} =$T_r\omega_n$.

TABLE II
RISE TIME OF THE 0/2 CRITICALLY DAMPED SECOND-ORDER DYNAMIC SYSTEM

ω_n (rad/s)	T_{r1} (s)	T_{r2} (s)	T_r (s)	K_{Tr}
1	0.5320	3.8900	3.3580	3.3580
2	0.2660	1.9448	1.6788	3.3577
3	0.1772	1.300	1.1227	3.3683
4	0.1330	0.9725	0.8395	3.3580
5	0.1063	0.7779	0.6716	3.3582
6	0.0886	0.6483	0.5596	3.3580
7	0.0760	0.5557	0.4797	3.3579
8	0.0665	0.4862	0.4197	3.3576
9	0.0591	0.4322	0.3731	3.3580
10	0.0532	0.3885	0.3353	3.3532

ω_n: System natural frequency
T_{r1}: System first rise time for 10 % time response.
T_{r2}: System second rise time for 90 % time response.
T_r: System rise time (T_{r2}-T_{r1}).
K_{Tr}: Rise time gain in: $T_r = K_{Tr}/\omega_n$

It is obvious from Table II that the gain K_{Tr} has very close values for the ω_n range investigated. The '*mean*' command of MATLAB [23] is used to provide the mean value of the gain KTs and the '*std*' command [24] is used to provide its standard deviation about its mean value. The two parameters for K_{Tr} are as follows:

$$K_{Tr-mean}=3.3585, St-Deviation_{-Tr}=0.00392 \qquad (7)$$

Delay Time, T_d:

The delay time of a step time response of a dynamic system is the time after which the time response reaches 50 % of its steady-state value [11]. With critically damped dynamic systems, this reduces to the intersection of the unit step time response with a 0.5 line. At t = T_d, Eq.2 gives:

$$(1+\omega_n T_d)\exp(-\omega_n T_d)-0.5=0 \qquad (8)$$

Eq.8 is a nonlinear equation in the dynamic system delay time. For a specific value of the system natural frequency, it is solved by the MATLAB command '*fsolve*' [22]. The application of '*fsolve*' to solve Eq.8 (giving the delay time T_d) for natural frequency, ω_n in the range $1 \leq \omega_n \leq 10$ rad/s. The results are given in Table III with the parameter K_{Td} ($T_d\omega_n$).

TABLE III
DELAY TIME OF THE 0/2 CRITICALLY DAMPED SECOND-ORDER DYNAMIC SYSTEM

ω_n (rad/s)	T_d (s)	K_{Td}
1	1.6783	1.6783
2	0.8392	1.6783
3	0.5595	1.6783
4	0.4196	1.6783
5	0.3356	1.6783
6	0.2797	1.6782
7	0.2397	1.6783
8	0.2098	1.6784
9	0.1865	1.6783
10	0.1678	1.6783

ω_n: System natural frequency
T_d: System delay time.
K_{Td}: Delay time gain in: $T_d = K_{Td}/\omega_n$

It is obvious from Table III that the gain K_{Td} has very close value to 1.6783 for the ω_n range investigated. The '*mean*' command of MATLAB [23] is used to provide the mean value of the gain K_{Td} and the '*std*' command [24] is used to provide its standard deviation about its mean value. The two parameters for K_{Td} are as follows:

$$K_{Td-mean} = 1.6783, St - Deviation_{-Td} = 0.000052 \quad (9)$$

Case Study 1:

To investigate the efficiency of the present procedure in defining the time-based characteristics of overdamped second-order systems of type 0/2 we consider a typical application from biomedical engineering where an I-first order, 1/2 compensators and a PD-PI controller were used to control the human blood pCO2 [25]. The author used the zero/pole cancellation techniques to assign some of the I-first order compensator parameters and came out with a standard 0/2 overdamped second-order transfer function for the closed-loop control system having 0.20833 rad/s natural frequency. The step time response of the control system for a unit step input (desired pCO2 change) is obtained using the '*step*' and '*plot*' commands of MATLAB [26] as shown in Fig.1.

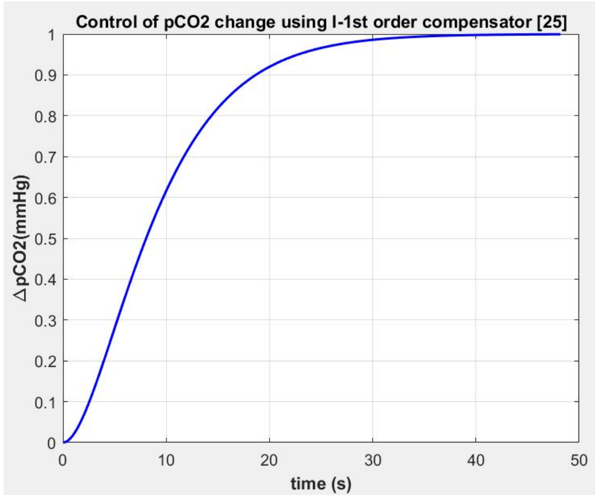

Fig.1 pCO2 step time response (case study 1).

The exact time-based characteristics of the control system step time response are obtained using the '*stepinfo*' command of MATLAB for rise and settling time [27] and the time response plot in Fig.1 for the delay time. The characteristic parameters (T_s, T_r and T_d) using the technique presented in this research work for critically damped second-order-like dynamic systems are obtained my dividing the derived characteristic gain in Eqs.4, 7 , 9 by the natural frequency of the dynamic system. The results are presented and compared in Table IV.

TABLE IV
NUMERICAL CHARACTERISTICS OF A 0/2 DYNAMIC SYSTEM (CASE STUDY 1)

Charact-erustic	**T_s (s)**	**T_r (s)**	**T_d(s)**
Exact	28.00418	16.12028	8.0567
Present Equation (T_i=K_i/ω_n)	4	7	9
Present value	28.0107	16.1213	8.0561
Error (exact – present)	-0.00652	-0.00108	0.00060
% Error	-0.0232	-0.0067	0.0074

T_i: Characteristic time parameter.
K_i: Characteristic gain parameter.
T_r, T_d, T_s: Rise, delay, settling times.

III. CRITICALLY DAMPED 1/2 SECOND-ORDER DYNAMIC SYSTEM

A lot of dynamic systems exhibit 1/2 overdamped second-order dynamic model characteristics in control engineering [28], power generation engineering [29], boiler engineering [30], automotive engineering [31], [32], marine engineering [33] and biomedical engineering [34]. The transfer function of a 1/2 second-order dynamic system, $G_{ds2}(s)$ is given by:

$$G_{ds2}(s) = \omega_n^2 (T_z s + 1) / (s^2 + 2\varsigma\omega_n s + \omega_n^2) \quad (10)$$

Where T_z is the time constant of the 1/2 second order dynamic system simple zero.

Using inverse Laplace transformation, the unit step time response of the 1/2 second order dynamic system defined by Eq.10 (with a unit damping ration for critical damping) is given by [35]:

$$c_2(t) = 1 - \exp(-at) + (a^2/b)[1-(b/a)]t\exp(-at) \quad (11)$$

Where: $a = \omega_n$ and $b = 1/T_z$.

The natural frequency ω_n in Eq.11 has vital effect on the step time response of the dynamic system. Therefore, it is essential to optimize its value for faster step time response without maximum overshoot. The results are casted in the form of a second-order polynomial determined by the author using MATLAB command '*polyfit*' [36] as follows:

$$\omega_n = 0.002493 T_z^2 - 0.0566086 T_z + 0.453383 \quad (12)$$

With 0.9995 correlation coefficient.

The time-based characteristics of the 1/2 dynamic system (T_s, T_r and T_d) are obtained using the same procedure applied to the 0/2 critically damped second order dynamic system. The results are presented in Table V for the time-based characteristics of the 1/2 critically damped second order dynamic systems for ω_n, T_s, T_d and T_r against the time constant T_z of the system simple zero in the range: $0.5 \leq T_z \leq 10$.

TABLE V
TIME-BASED CHARACTERISTICS OF THE CRITICALLY DAMPED 1/2 SECOND-ORDER DYNAMIC SYSTEM

T_z (s)	ω_n (rad/s)	T_s (s)	T_d (s)	T_r (s)
0.5	0.420	13.0125	3.4769	7.8874
1	0.485	13.1721	3.0834	7.8355
2	0.350	13.2062	2.6915	7.9566
3	0.310	13.4446	2.3997	7.5747
4	0.266	13.7067	2.4459	7.3589
5	0.230	14.1063	2.6043	8.1038
6	0.201	14.3211	2.8586	8.7757
7	0.180	15.0028	3.0079	9.0535
8	0.162	15.6149	3.2373	9.6158
9	0.148	15.9974	3.4346	10.0595
10	0.135	16.9764	3.7077	10.7817

T_z: 1/2 system zero time constant
ω_n: 1/2 dynamic system natural frequency.

Each time-based characteristic is related to ω_n through the relation K_{ij}/ω_n as we did with the 0/2 overdamped second-order system where K_{ij} is the gain corresponding to each time-based characteristic parameter. The values of K_{ij} for settling time, delay time and rise time against T_z is given using data in Table V and presented in Table VI.

TABLE VI
GAIN PARAMETER OF THE TIME-BASED CHARACTERISTICS OF THE CRITICALLY DAMPED 1/2 SECOND-ORDER DYNAMIC SYSTEM

T_z (s)	K_{Ts}	K_{Td}	K_{Tr}
0.5	5.4652	1.4603	3.3127
1	5.3347	1.2488	3.1734
2	4.6222	0.9420	2.7848
3	4.1678	0.7439	2.3482
4	3.6460	0.6506	1.9575
5	3.2444	0.5990	1.8639
6	2.9072	0.5717	1.7551
7	2.7005	0.5414	1.6296
8	2.4305	0.5244	1.5577
9	2.3676	0.5083	1.4881
10	2.2918	0.5005	1.4555

K_{Ts}: Settling time gain parameter.
K_{Td}: Settling time gain parameter.
K_{Tr}: Settling time gain parameter.

It is obvious from Table VI that the time constant gain Kij has a decreasing nature with the zero time constant Tz. Therefore, a polynomial model is recommended for this variation as follows:

For settling time:

$$K_{Ts} = 0.0330783 T_z^2 - 0.691855 T_z + 5.893862 \quad (13)$$

With 0.9991 correlation coefficient.

For delay time:

$$K_{Td} = 0.0003637 T_d^4 - 0.0106717 T_d^3 + 0.1171805 T_d^2 - 0.5913639 T_d + 1.7305581 \quad (14)$$

With 0.9999 correlation coefficient.

$$K_{Tr} = -0.00017514 T_r^3 + 0.0539803 T_r^2 - 0.5913639 T_r + 3.6591019 \quad (15)$$

With 0.9967 correlation coefficient.

Case Study 2:

To investigate the efficiency of the present procedure in defining the time-based characteristics of critically damped second-order systems of type 1/2 we consider a dynamic system with a model defined as a 1/2 critically damped second-order system having T_z = 5.5 s. The technique presented in the present work is applied as follows:

- First of all the optimal natural frequency is assigned using Eq.12 as:

$$\omega_n = 0.21745 rad / s \quad (16)$$

- The settling time, delay time of the dynamic system is obtained using Eqs.13, 14 and 15 respectively and given by:

$$T_s = 14.20672, T_d = 2.66778, T_r = 8.2229 s \quad (17)$$

- The unit step time response of the dynamic system is obtained using Eq.10 for unit damping ratio, 5.5 s zero time constant and natural frequency of Eq.16 using the MATLAB '*step*' command [26] as shown in Fig.2.

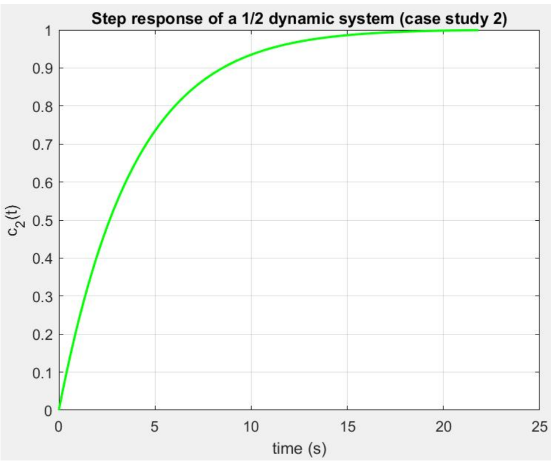

Fig.2 Step time response of a 1/2 dynamic system (case study 2).

The exact time-based characteristics of the control system step time response are obtained using the '*stepinfo*' command of MATLAB for rise and settling time [27] and the time response plot in Fig.2 for the delay time. The characteristic parameters (T_s, T_r and T_d) using the technique presented in this research work for overdamped second-order-like dynamic systems are obtained by dividing the derived characteristic gain in Eqs.13, 14 , 15 by the natural frequency of the dynamic system. The results are presented and compared in Table VII.

TABLE VII
NUMERICAL CHARACTERISTICS OF A 1/2 DYNAMIC SYSTEM (CASE STUDY 2)

Charact-erustic	**T_s (s)**	**T_r (s)**	**T_d(s)**
Exact	13.8768	2.6380	8.1116
Present Equation (T_i=K_i/ω_n)	13	14	15
Present value	14.2067	2.6678	8.2229
Error (exact – present)	-0.32992	-0.0298	-0.1113
% Error	-2.3775	-1.1296	-1.3721

IV. CONCLUSIONS

- This research paper investigated a novel evaluation procedure for the characteristics of critically damped second-order-like dynamic systems.
- The characteristics covered: settling time, delay time and rise time.
- The work is unique for critically damped second-order-like dynamic systems of 0/2 and 1/2 types.
- The objective was to define the specific characteristic in the form of K_{ij}/ω_n.
- The gain K_{ij} had a unique value for each of the characteristic elements for type 0/2 critically damped second-order- system independent of the natural frequency of the dynamic system.
- The dynamics of the 1/2 critically damped second-order system were function of the time constant of its simple zero and its natural frequency. Because of which the research work found an optimal value for the system natural frequency leading to a minimum settling time.
- For the 1/2 critically damped second-order system the gain K_{ij} was function of the time constant of the system simple zero. Curve fitting techniques were applied to fit a reasonable polynomial for the characteristic gain.
- Two case studies were presented for each type of the investigated second-order dynamic systems. The time-based

characteristics were compared between the exact characteristic values and the evaluated ones using the derived polynomial models. The maximum difference was 0.023 % for the first case study and 2.37 % for the second case study.

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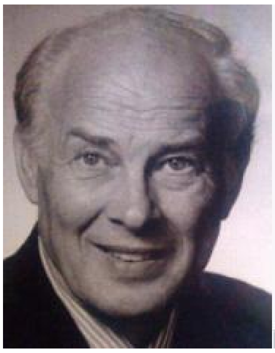
DEDICATION

Late Prof. JOHN PARNABY



- He was the chairman of the ‘Industrial Engineering Department’ of Bradford University during 1970’s.
- He supervised my Ph.D. research work during 1974-1979.
- He was a great professor and a major scientist and inventor in industrial engineering and automatic control.
- He taught me ‘automatic control’.
- He died in 5 January, 2011.
- Thanks Prof and I have the honour to dedicate this research work to you.

BIOGRAPHY

GALAL ALI HASSAAN

- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his B.Sc. and M.Sc. from Cairo University in 1970 and 1974.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
- Now with the Faculty of Engineering, Cairo University, EGYPT.
- Research on Automatic Control, Mechanical Vibrations, Mechanism Synthesis and History of Mechanical Engineering.
- Published more than 360 research papers in international journals and conferences.
- Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.
- Honourable Chief Editor of the International Journal of Computer Techniques.
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